



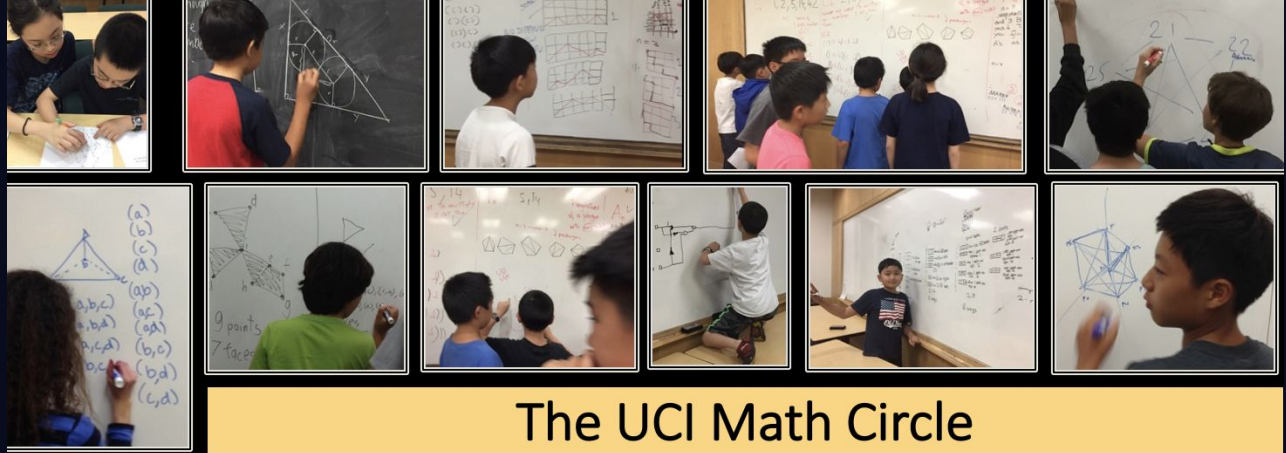
The 2020-2021 UCI Math Circle: Building an Online Community of Young Math Researchers

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Texas A&M University

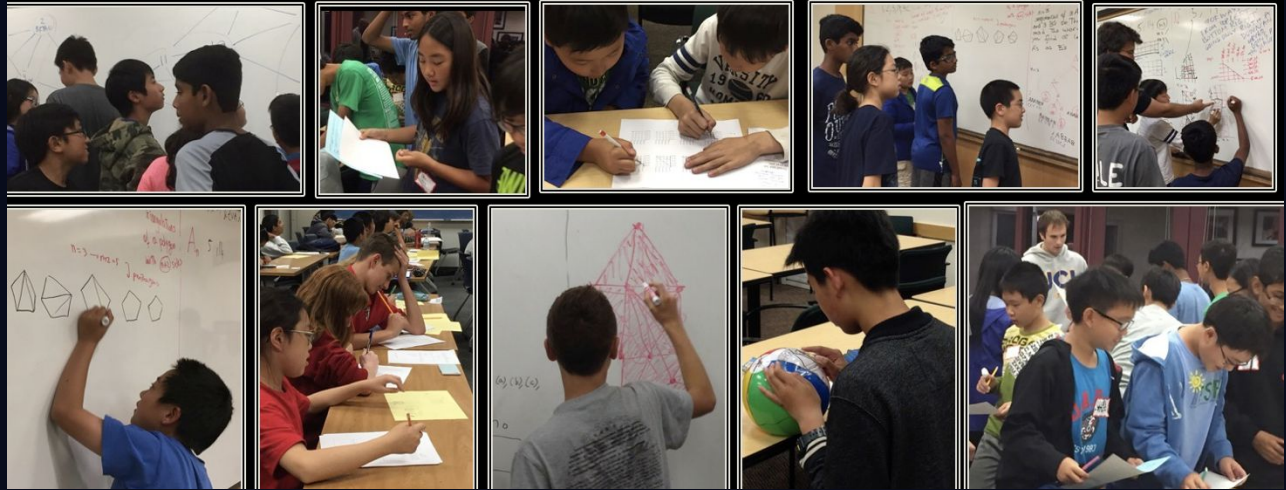
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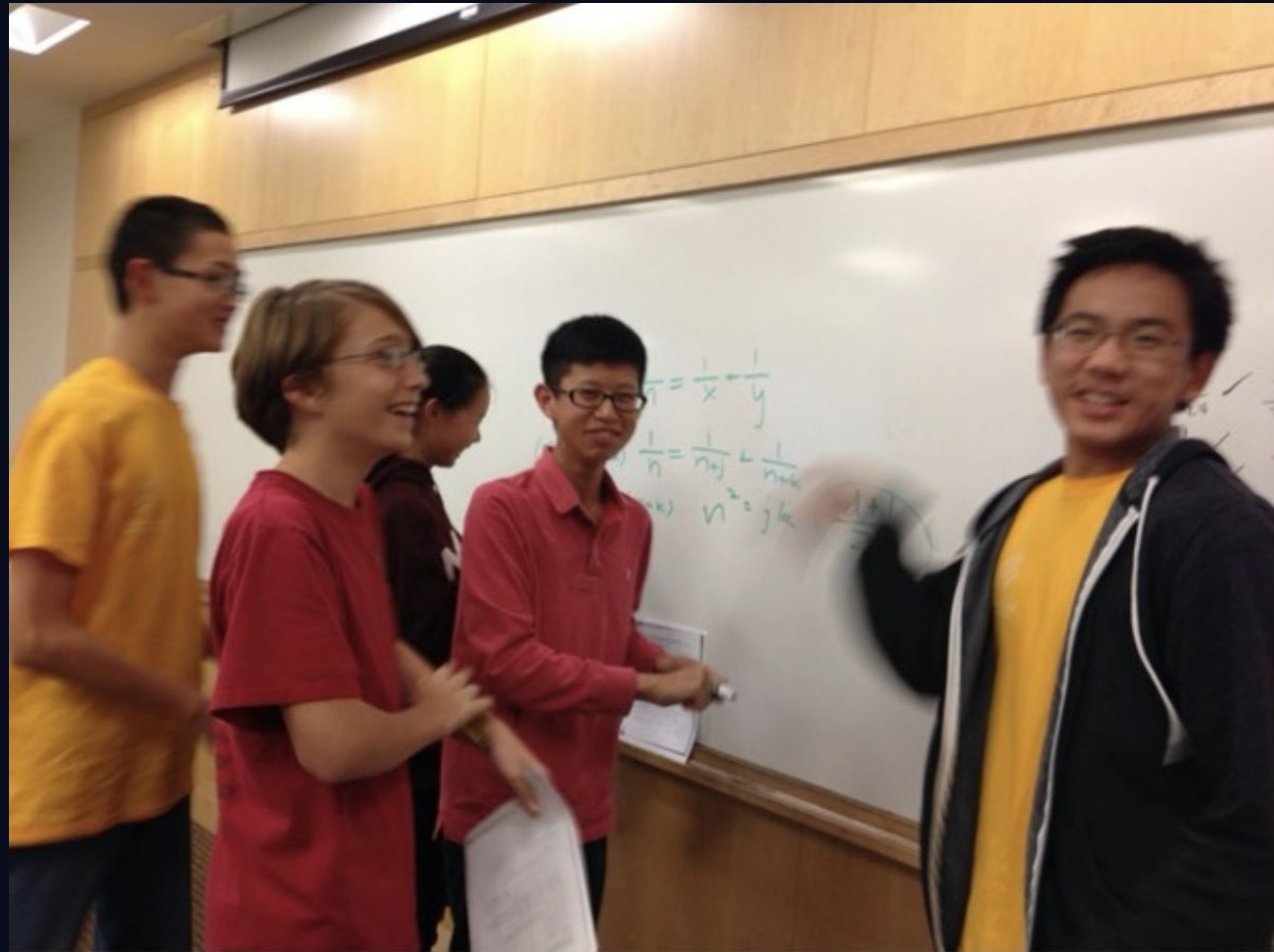
MathFest - 8/5/22

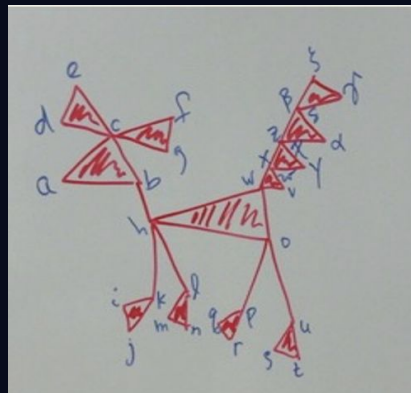
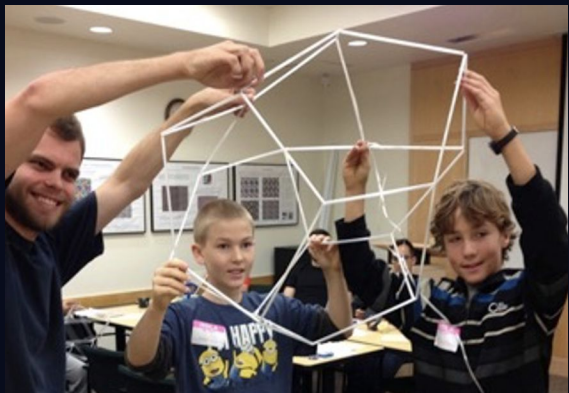
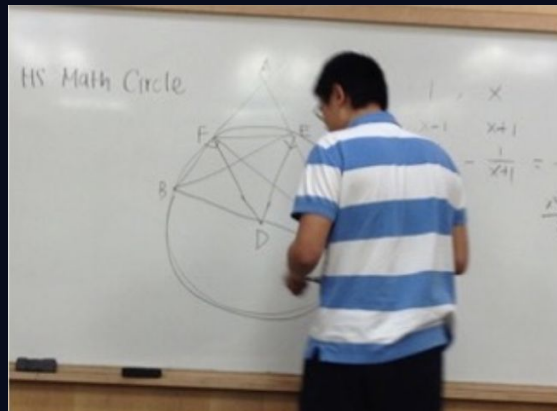
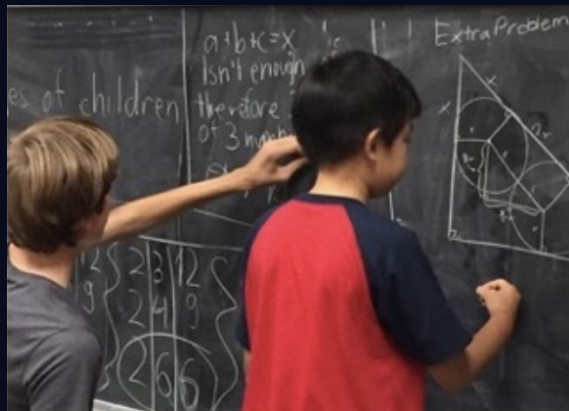


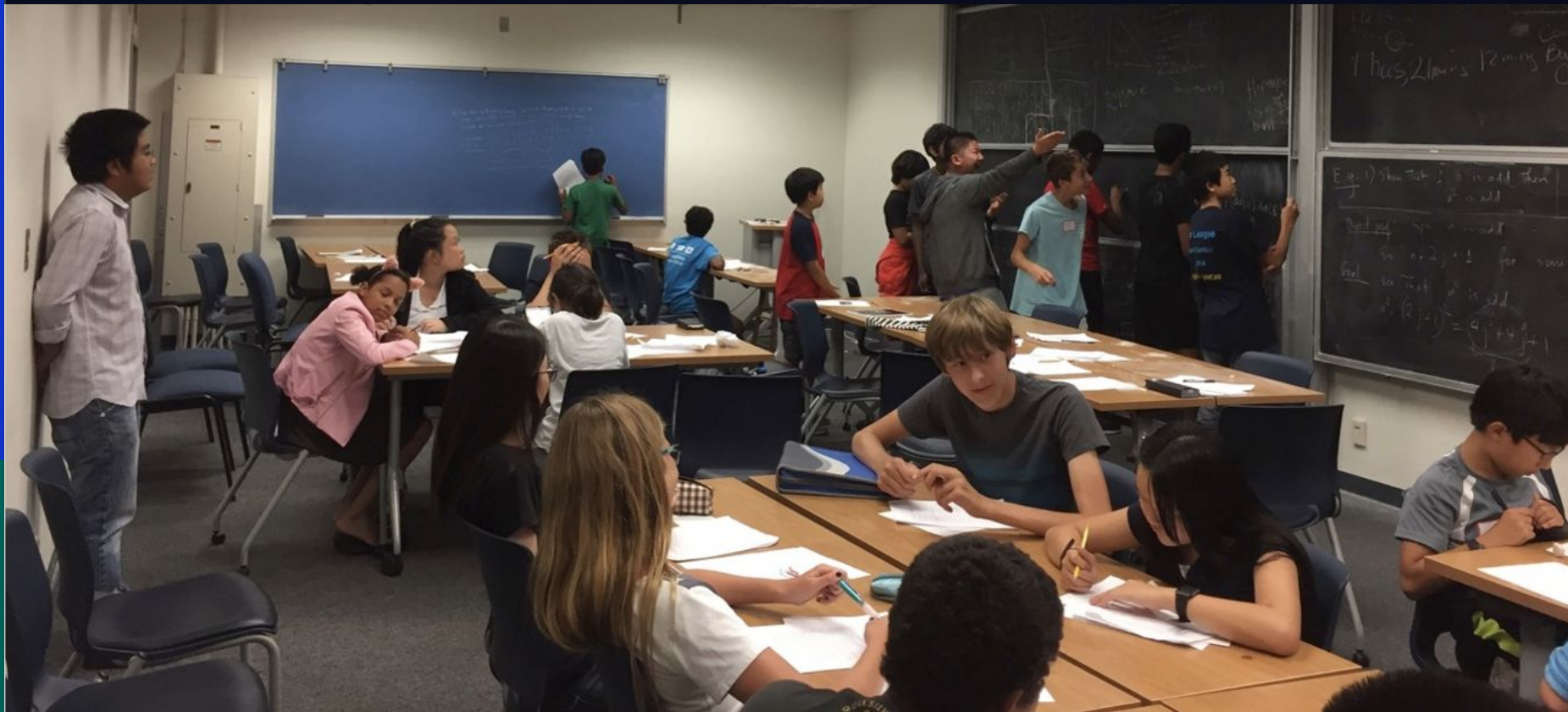
The UCI Math Circle



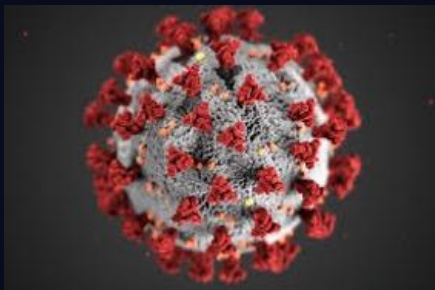
The UCI Math Circle
has been spreading
the joy of
mathematics
since 2012







Until Winter 2020, all meetings of the UCI Math Circle were in person, at UCI



At the start of the pandemic, in Spring 2020, the UCI Math Circle took a break (for the first time after 23 quarters)



We resumed our meetings online in Fall 2020



In 2020-2021 the Math Department funded 2 graduate students (given the title of UCIMC Coordinators) with TAships to facilitate the transition



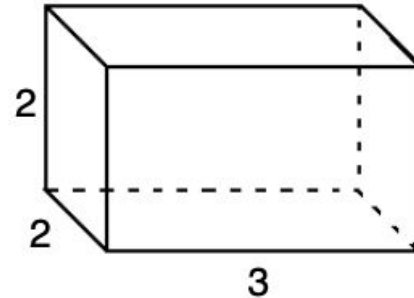
New UCIMC Coordination

	Fall 2020 - Spring 2021	Fall 2012 - Spring 2020
Select curriculum for weekly sessions	Nick and Yasmeen	Weekly MC leader (level I & II)
Train mentors for a weekly session	Nick and Yasmeen	N/A
Run a weekly session	(Nick and Yasmeen) + several mentors	Weekly MC leader + an assistant
Email participants & parents	Nick and Yasmeen	Alessandra
Recruit mentors (session leaders)	(Nick and Yasmeen) + Alessandra	Alessandra
Math competitions (e.g., AMC 10/12)	Alessandra + a grad student/postdoc	Alessandra + a grad student/postdoc
Others (e.g., parent consent, donations)	Alessandra	Alessandra
Advice and oversight	Alessandra	Alessandra



In-person math circle lessons

- Curriculum varied week-to-week; was entirely determined by the instructor
 - Instructors would spend hours on content creation
 - Instructors would usually volunteer once a quarter

Problem 2.7 *Given the rectangular prism shown below, show that the geometric mean of the side lengths equals the side length of the cube that has the same volume as the rectangular prism.*



An arithmetic-geometric mean (AM-GM) problem from an in-person math circle



Curriculum and Pedagogical Choices for the Online UCI Math Circle

Planning for an Online Math Circle

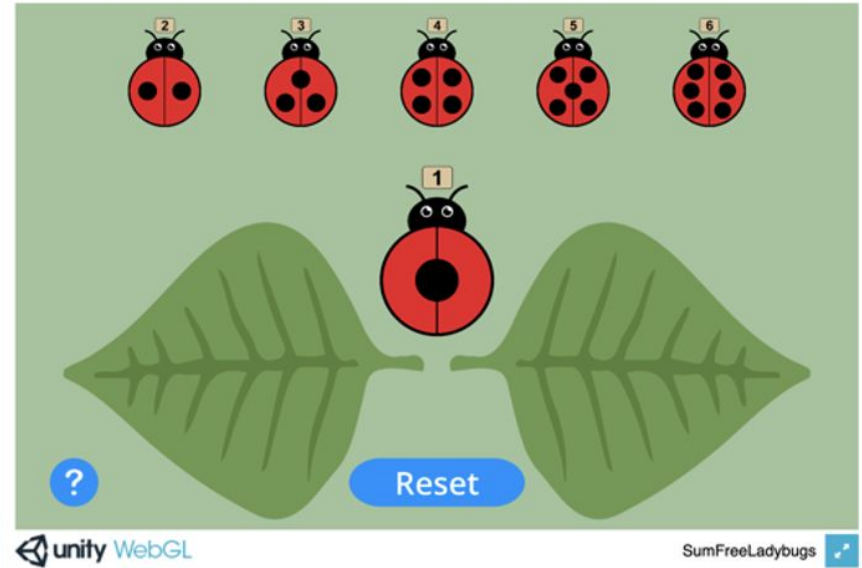
- Foster engagement with the material and create a fun, enriching, and supportive environment.
- Provide opportunities for students to share their work and for instructors to give feedback.
- Create opportunities for socializing, similar to how one could “talk to a neighbor” in person



An instructor (center, seated) reads over the work of the students around him

The online UCIMC curriculum

- Nick and Yasmeen would select the curriculum for every lesson.
- With the permission of the Julia Robinson Math Festival (JRMF) we used their applications/lessons because they are
 - Very interactive
 - Applications are easy to use and understand
 - Often secretly are based on deep math in group theory, combinatorics, probability and can be explored at many levels

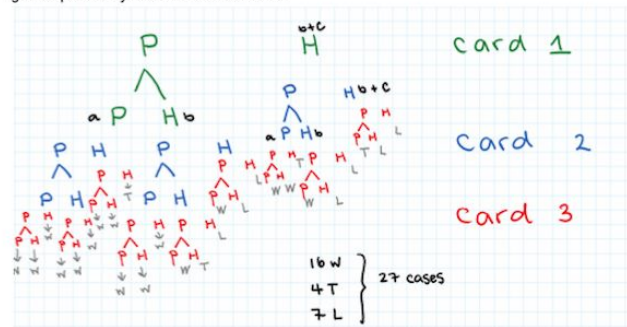


Screenshot of a JRMF application “Ladybugs”

The Cheat Sheet

- Nick and Yasmeen met weekly for two hours to play a JRMF activity and design a lesson
- Each meeting they created a cheat sheet
 - Questions for volunteers to ask the students
 - Answers to the questions
 - Tips for mentors (volunteers)

get the probability of one of the outcomes.



12. N/A.

13. Label the tiles as A B C and the sides of the tiles as A1, A2, B1, B2, C1, C2. The possibilities for what we may see is the following:

- A1 A2
- A2 A1
- B1 B2
- B2 B1
- C1 C2
- C2 C1

Each possibility is equally likely; thus each event occurs with probability $\frac{1}{6}$ times. Events a and b result in seeing two penguins. Results C and D results in seeing one penguin. Thus, the expected number of penguins is therefore

$$2 \left(\frac{1}{6} + \frac{1}{6} \right) + 2 \left(\frac{1}{6} \right) = 1.$$

Thus, we expect to see 10 penguins after we draw 10 tiles.

14. Label the tiles as A B C and the sides of the tiles as A1, A2, B1, B2, C1, C2, as shown above. The possibilities for what we may see is the following:

- A1 A2
- A2 A1
- B1 B2
- B2
- C1
- C2

Each possibility is equally likely; thus each event occurs with probability $\frac{1}{6}$ times. Events a, and b result in seeing two penguins. And C results in seeing one penguin. The expected number of penguins is therefore

A page from the cheat sheet

Intricacies of the Cheat Sheet

1. **[For the younger students]** Have them list out the prime numbers between 1 and 20.
2. Can a game of prime cubes use all 8 numbers? If not, what is the longest game? How

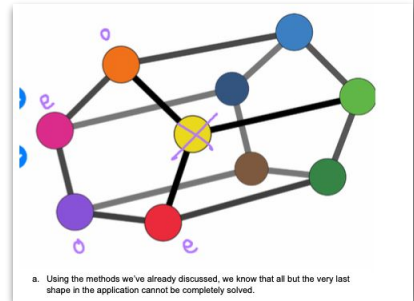
Age appropriate questions

13. Can you come up with any necessary conditions for solving the 6-prism?

Suggested open-ended, no-correct-answer problems for further exploration

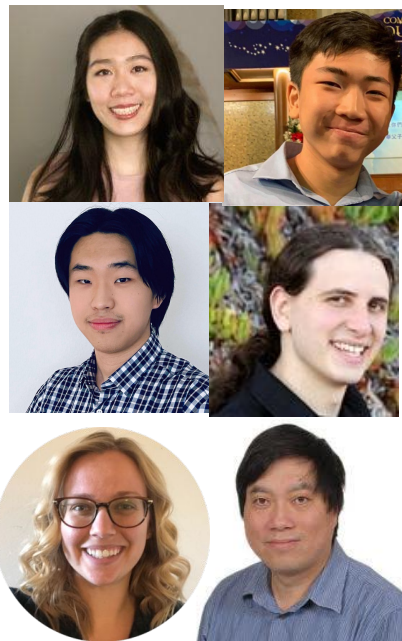
iii)
$$C_n = B_{\{n-1\}} + 1 + B_{\{n-1\}} + 1 + B_{\{n-1\}} + 1 + B_{\{n-1\}} = 4(2^{n-2}) + 3$$
$$= 2^{n+2} - 5 \text{ valid for } n > 1.$$

In depth explanations of the math behind the activity for the mentors



The mentors

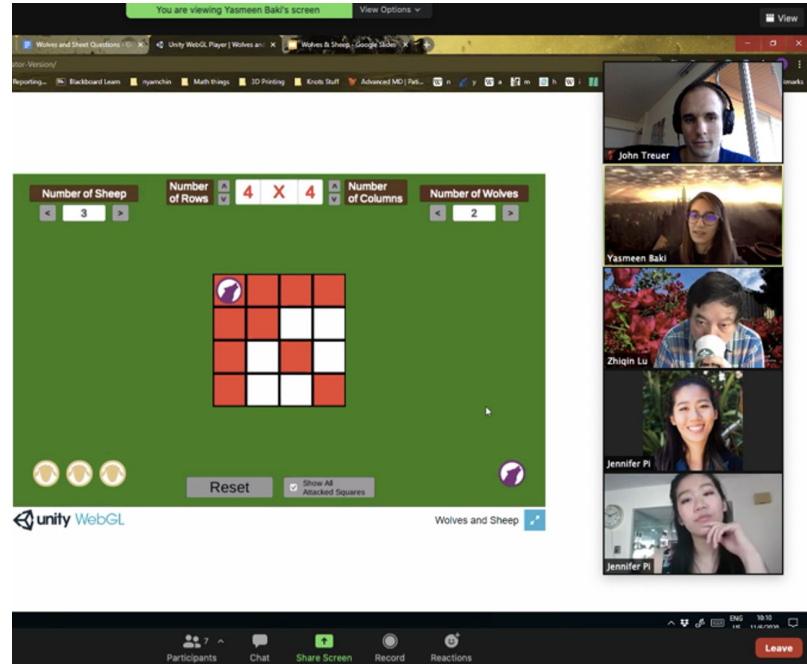
- No longer needed volunteers to design content
- But needed several to help facilitate the meeting
- Recruited volunteers: math Ph.D. students, faculty and undergraduates with teaching experience as Learning Assistants
 - Asked them to volunteer for the entire quarter
 - Gave them the title “mentor”
 - Required them to attend a weekly mentors’ meeting
 - Total time commitment for them ~2 hours per week



A few of the UCIMC mentors

The Mentors' Meeting

- Each Friday, Nick and Yasmeen held a one hour meeting with the mentors
- Simulated the upcoming activity with the mentors playing the role as the students:
 - Practiced activities with them
 - Asked for observations, hypotheses about the activities
 - Steered the discussion by asking questions from our cheat sheet



On the right: The two UCIMC Coordinators and two mentors

The Mentors' Meeting

- At the end, we revealed the week's cheat sheet
 - showed them the questions they had and had not answered.
- We ran the mentor meeting this way to
 - Simulate the upcoming activity
 - Show by example how we would like the mentors to interact with the students
 - As a bonus, we could explore the underlying math at a deeper level than the K-12 students

The screenshot shows a Zoom meeting window. The main content is a game interface titled 'Wolves and Sheep' by unity WebGL. The game board is a 7x7 grid with a diagonal line of red squares. The interface includes controls for 'Number of Sheep' (set to 2), 'Number of Rows' (7), 'Number of Columns' (7), and 'Number of Wolves' (3). Handwritten purple annotations include the formula $n^2 - 2(n-1) - n$ and a summation $2 \sum_{k=1}^{n-2} k$. A vertical bracket on the left is labeled $n-2$, and a horizontal bracket at the bottom is labeled $(n-2)(n-1)$. The Zoom meeting controls at the bottom show 'Stop Video', 'Participants', 'Chat', 'Share Screen', 'Record', 'Reactions', and a 'Leave' button.

The mathematical discussions usually go deeper than what we would expect of the students



A Typical Meeting

The UCIMC Meeting

- Nick or Yasmeen took turns leading the UCIMC meeting
 - All students got to know Nick and Yasmeen
 - Constant point of contact for the students: every week one of them welcomed all to UCIMC
- On arrival a fun fact or a short problem was displayed
 - Some students may not arrive exactly on time so this gave the early students something to think about



Students and mentors thinking about the opening slide

A presentation slide with a blue and green border. The title "Welcome to Math Circle!" is in brown. Below it, the text reads: "Before we start today's activity, think about how you could solve the following problem... Six positive integers are picked between 1 and 2020 at random. What is the probability that some pair of these integers has a difference that is a multiple of 5?". At the bottom right, there is a fun fact: "Did you know? The Pigeonhole Principle can be used to solve all kinds of interesting problems, and it often appears in solutions to competition math problems. One famous application argues that there must be two people in London who have the same number of hairs on their heads." The slide is decorated with blue and green diagonal lines in the corners.

The opening slide

Challenge Question:

- At the end of the previous week's math circle, a challenge question based on the activity to try at home was given
 - We encouraged them to email us their solution
- After the opening slide, we reviewed present the submitted answers

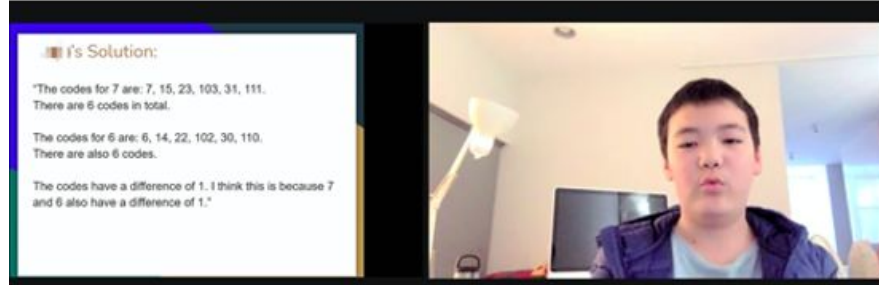
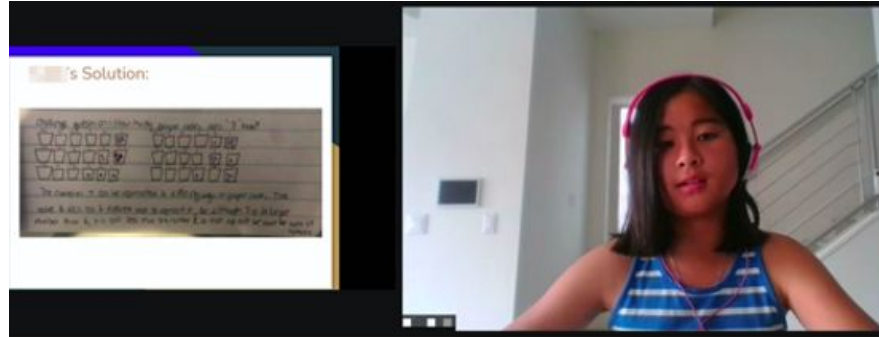
Last Week's Challenge Question:

Create a ring with exactly 3 starting numbers that has more than one solution. Try to make your ring have as few arrows as possible.

A sample challenge question from Winter 2021.

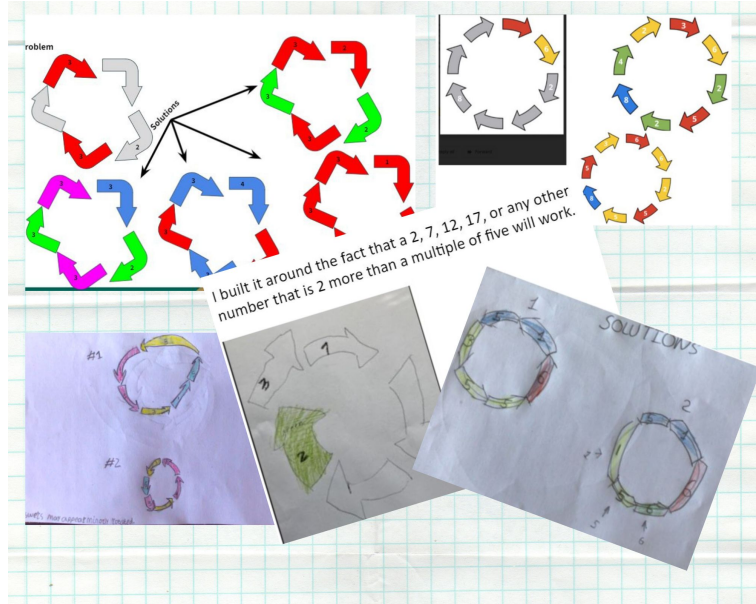
Students loved sharing their answers

- We gave each student an opportunity to say something about their work
- We asked them a question to encourage them to speak. E.g
 - What was your favorite part about working on the question?
 - What was the most surprising thing you found?



Two students present their solutions to the challenge question

A collage of answers



A collage of six student answers to one week's challenge question

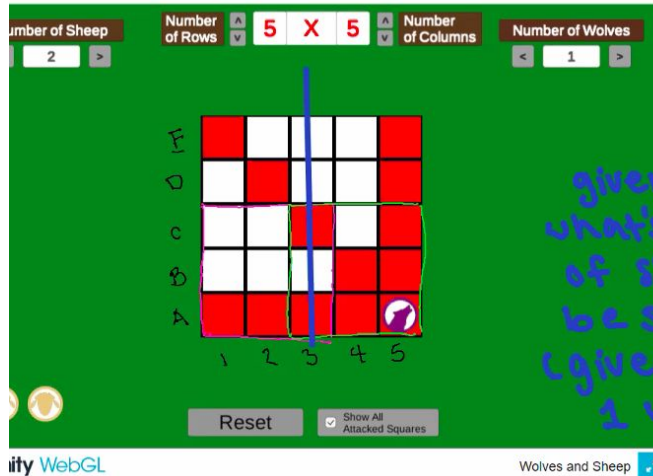
- All student answers are presented - both right and wrong
- We never criticized any of the answers.
 - They became very comfortable with sending in answers
- Yasmeen and Nick never presented a “correct answer” to any of the problems.
 - Only the student answers were shown.

Breakout rooms open

- While one coordinator was leading the session, the other one was busy preparing the breakout rooms.
- Breakout rooms were labeled by the names of the mentors and the grades they worked with
 - “Yasmeen - Grades 9-12”
 - Mentors worked with the same grade levels (more or less) every time
- We allowed the students to choose their breakout room so that
 - more advanced kids could work with older students
 - students could work with mentors they liked
 - since mentors worked the same breakout room each week, students who joined that room could get to know the mentor and vice versa.

Mentors begin the activity!

- Mentors used their cheat sheet.
- Mentors direct their breakout rooms similar to the way the mentor meeting was held
- Over the course of the quarter, the students and mentors bonded.



$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$
row, column
~ Remote Control?

given $[n \times n]$ board,
what's maximum #
of sheep that can
be safely placed?
(given that exactly
1 wolf is placed)

Yasmeen working with the grades 9-12 breakout room

Goodbye and a new challenge

- With five minutes left, everyone returned to the main room and the leader of the session (Nick or Yasmeen) closed the session by
 - asking students what they discovered
 - showing the new challenge question

Link to Digital Application: <https://www.jrmf.org/digitalapps/penguin-island>

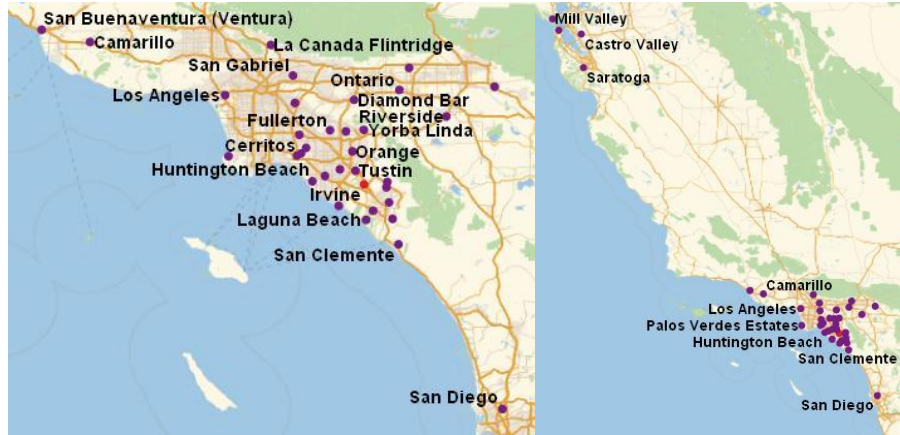
Suppose we have a tile limit of 2 and use all 7 different kinds of tiles (go to options > + > change all of the zeros to ones). If you use the strategy of flipping every tile, what is the probability that you will win?

The new challenge question. The new challenge questions were later emailed to the students for their convenience.



Positive Reception

Increase in Geographic Reach

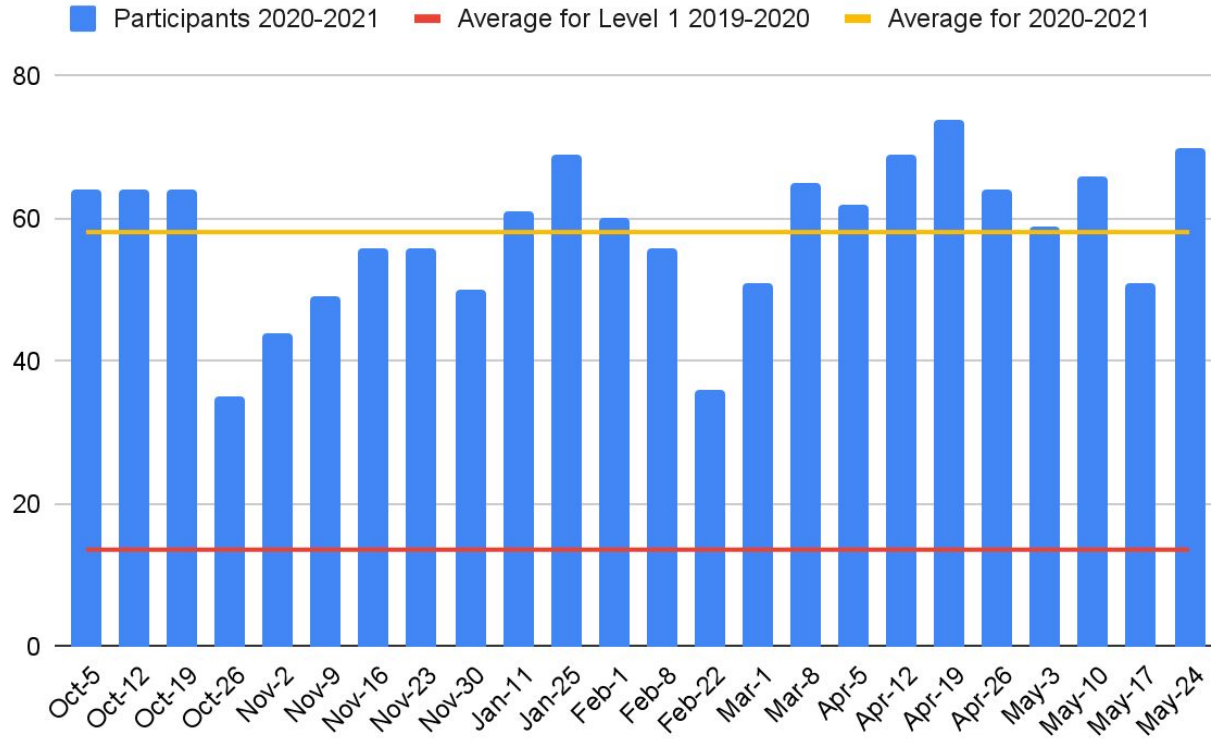


California cities represented in 2020-2021 registration data.

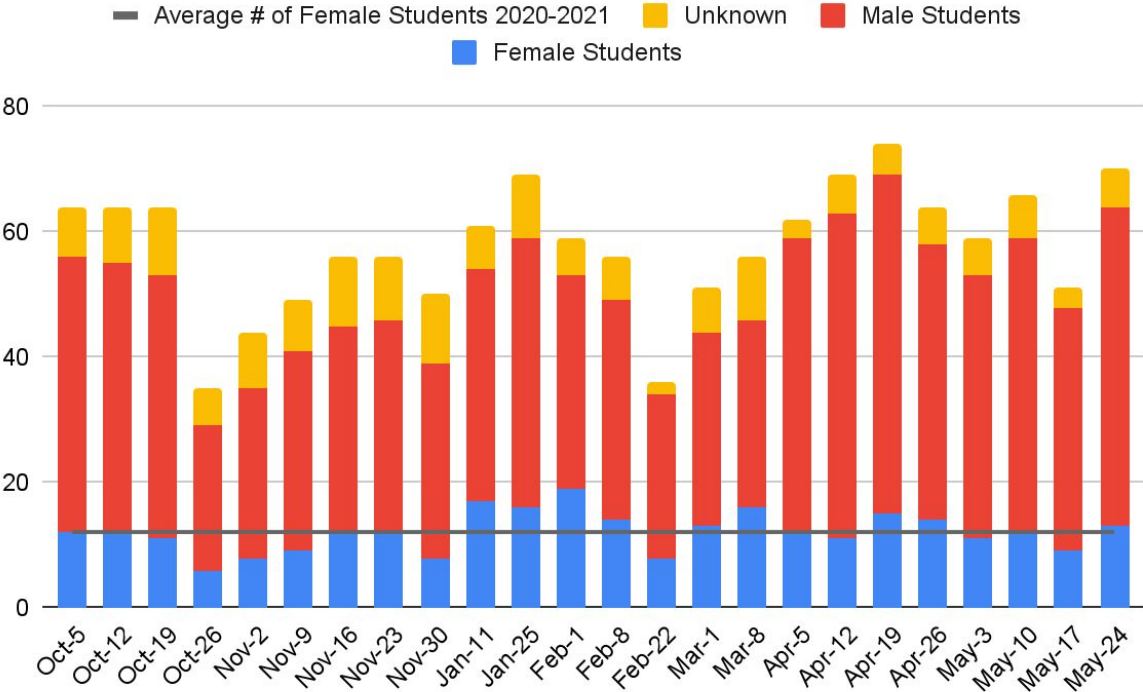


Locations of 44 students surveyed during one 2020-2021 meeting.

2020-2021 Attendance



Gender diversity among 2020-2021 participants





Feedback

Feedback - Participants

“[UCIMC] teaches you how to think like a mathematician”

“Everyone can always participate by sharing a mathematical fact or formulas to help the class along.”

“I loved how [UCIMC] goes deep into the abstract thinking of math, and makes us think of different patterns.”

“I like the way the interactives explain the topic in simpler terms.”

Feedback - Mentors

"[UCIMC's meetings] were the only Zoom meetings I actually looked forward to."

Teaching gains:

- Learn "patience and delicateness"...
- how to keep students engaged
- how to ask thought-provoking questions
- how to explain things in a less technical but more intuitive way

Favorite aspect: Working with bright young students

Another very positive aspect: The opportunity to interact with fellow members of the Math Department during the COVID-19 pandemic

"It was nice to casually chat with people I would have not had the chance to meet otherwise."

"I've always loved teaching, but Math Circle made me feel like the educators had a community. This is not something I always feel when teaching my [college] courses."

Feedback - Parents

We asked parents of students who consistently attended for feedback:

“We love that it brings a playful and social aspect...Luke has felt very isolated from other kids his age in this area of his life. It is hard to find a ton of kids that love math like he does—but he has found this here with this great group!”

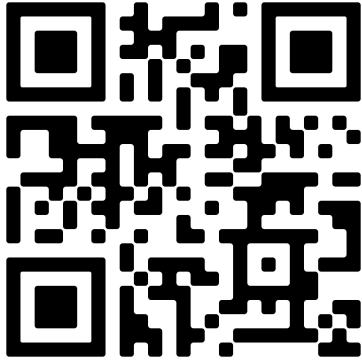
“An opportunity to have social interaction focused around math and led by young people, students, who are also interested in math and lead them by that example, was simply incredible”

“Flowing and very very engaging. The fact that there are two student instructors in every breakout room worked out so well!”

fun math group
 creative
 hands-on
 fun math activities
 it is good
 learn to think
 special
 electrifying
 fun math circle
 fun cool puzzles
 smart
 thought-provoking
 fun math group
 entertaining
 logical puzzles
 is very good
 it is awesome!
 unique
 interactive
 one interesting hour
 very fun program
 exciting activities club
 nice
 articulative
 good
 amusing
 different but fun
 awesome
 stimulating
 active
 math
 pattern analyzing
 enjoyable
 inquiring
 amazing
 different
 fun math activities
 intellectual
 sometimes a little hard
 cool
 welcoming
 fun math activities
 fun math circle
 pretty dang fun
 fun math circle
 fun cool puzzles
 smart
 fun math activities
 intellectual
 sometimes a little hard
 cool
 welcoming
 fun math activities
 fun math circle
 pretty dang fun
 fun math circle
 fun cool puzzles
 smart

"Describe the UCI Math Circle using only three words"

Thank you!



Our paper in the
Journal of Math Circles

UCIMC participation increased when it went online

- No geographic restrictions from participants
- No need for parents to drive to UCI and spend money on parking
- It's possible that fewer alternatives were available to youth during the pandemic
- Strong sense of community (same mentors throughout the quarter/year)
- More agency for students (breakout rooms offered several options for mentors/level)
- JRMF curriculum is excellent (low-floor/high ceiling, open-ended problems)
- An enthusiastic parent advertised the program on social media (to parents of mathematically talented youth in Ventura County, near Santa Barbara)

Participation from girls increased in the online UCIMC

- Constant presence of at least two, but often more, female mentors
- “The open-ended problems and activities contributed to a learning environment which is inclusive and welcoming for all.”
- The JRMF exploratory curriculum made the experience less competitive, more collaborative and also more fun (*many activities are presented as games*)
- Mentors volunteered for an entire quarter. Time to build connections with mentors.
- Small breakout rooms with mentors moderating the conversations allowed for a calmer learning environment

[In-person] *“it is difficult to be a girl around all too active boys.”*

[Online] *“It doesn’t feel like the boys are dominating the conversation and that feels a lot more inclusive.”*

Online safety at UCIMC

- Never had any incidents
- We collected parental consent forms
- Parents were welcome to join
- A faculty member was present at all of the meetings
- No private student-to-student chat allowed
- Students could chat with everyone, or just with the mentors

What happened to UCIMC in 2021-2022?

- At the end of Spring 2021, we surveyed all UCIMC participants
- 72.7% stated they would prefer that UCIMC remain online
- In particular, *all the students who live outside Orange County* (but 1) wanted UCIMC to remain online because it would be impossible for them to attend an in-person math circle at UCI.
- *65% of the students who live in Orange County* said they would prefer an online format because it did not require transportation to UCI. Others mentioned they were nervous about returning to in-person activities.
- Because of this feedback, UCIMC remained online online in the 2021-2022 academic year.

How was UCIMC funded? (2020-2021)

- UCI Department of Mathematics funding
 - Nick and Yasmeen were given departmental TAship.
 - Working as UCIMC Coordinators covered half of their Teaching Assistantship duties as UCI Math Ph.D. students
- Donations from parents
 - Emails soliciting donations were sent to the parents.
- Leftover fees from math competitions
 - UCIMC annually holds several math competitions: AMC 8, 10, 12 and Tournament of the Towns
 - Money not used to administer the tests was used to help fund UCIMC